

On exactness properties preserved under pro-completions of finitely complete categories, and why this is useful

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As the theory of approximate operations shows, an abstract regular category with finite colimits has an inherent algebraic semantics that allows one to automate generalization of certain universal-algebraic arguments from the context of a variety having a given exactness property to the context of such a category. Since colimits play a crucial role for expressing this semantics, a question arises whether any regular category having an exactness property can be embedded in the one which has finite colimits and the same exactness property. In the literature, a few instances have been known of the fact that an exactness property is preserved under the so-called pro-completion of a finitely complete category, which is always cocomplete. A few years ago, in a joint work with P.-A. Jacqmin, we identified a wide class of exactness properties for which a general preservation under pro-completion result can be obtained. In an attempt to understand the proof of this result in conceptual terms, we arrived to two interesting observations, among others:

- (1) the language of sketches seems to provide an elegant approach in formalizing the definition of an exactness property in general;
- (2) the proof of the result is essentially a calculation in the 2-category Cat , involving comma objects and the Beck-Chevalley condition arising from point-wise Kan extensions.

The aim of this talk to is explain these observations, to show the relevance of our result back in the theory of approximate operations and it's use in working with categories defined by exactness properties, as well as to pose a couple of naturally arising open questions in this area.